Renormalization group in $2N$-component fermionic hierarchical model

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We consider $2N$-component fermionic model on the hierarchical lattice. It is well known that hierarchical models of statistical physics may be considered as discrete versions of p-adic field models. For review of the results for 4-component model see [1].

The Gaussian part of the hamiltonian of the model is invariant under renormalization group transformation with parameter α, and self-interaction is given by the free measure density function:

$$f(\psi) = c_0 + \sum_{k=1}^{N} \frac{c_k}{k!} \left(\psi^+\psi^-\right)^k,$$

where $\psi^+\psi^- = \sum_{k=1}^{N} \psi_k^+\psi_k^-$, $\psi_1^+, \psi_1^-, \ldots, \psi_N^+, \psi_N^-$ are generators of Grassmann algebra.

Block-spin renormalization group (RG) transformation $R_{\alpha,n}$ is computed explicitly:

$$R_{\alpha,n} w_t = \sum_{j=t}^{N} (-1)^{j-t} n^{\alpha(N-j)} \binom{N-t}{N-j} \sum_{j_1+\cdots+j_n=j} \frac{j!}{j_1!\ldots j_n!} \prod_i^n w_{j_i}, \quad t = 0, \ldots, N,$$

where variables w_k are related to c_k by the equations:

$$w_j = \left(1 - n^{\alpha-1}\right)^j \sum_{s=0}^{N-j} \binom{N-j}{s} c_s (-1)^{N-j-s}, \quad j = 0, \ldots, N.$$

Let $v_j = w_j/w_0$. Fixed points of RG-transformation may be found as solutions of some system of algebraic equations on $(v_1, \ldots, v_N)$. For example, in the case of $N = 3$ we have the following system:

$$nv_1 = n^\alpha \frac{v_1 + 2v_2 + v_3}{1 + 3v_1 + 3v_2 + v_3}, \quad nv_2 + n(n-1)v_1^2 = n^{2\alpha} \frac{v_2 + v_3}{1 + 3v_1 + 3v_2 + v_3},$$

$$nv_3 + 3n(n-1)v_1v_2 + n(n-1)(n-2)v_1^3 = n^{3\alpha} \frac{v_3}{1 + 3v_1 + 3v_2 + v_3},$$

which has several nontrivial solutions.

An interesting relation between RG-transformations in the fermionic and bosonic models is found. It turns out, that one transformation coincide with another when N is replaced with $-N$.

References

1. M.D.Missarov, *Renormalization group solutions in fermionic Dyson model,* Asymptotic combinatorics with applications to mathematical physics, V.A.Malyshev and A.M.Vershik (Eds.), Kluwer Academic Publishers, 2002, pp 151-166.