On p-adic Potts model with competing interactions on the Cayley tree

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Recall that the Cayley tree Γ^k of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\Gamma^k = (V, L)$, where V is the set of vertexes of Γ^k, L is the set of edges of Γ^k. Two vertices $x, y \in V$ are called *neighboring*, and denoted by $< x, y >$ if there is an edge $l \in L$ with its endpoints x, y. The distance $d(x, y), x, y \in V$, on the Cayley tree, is the length of the shortest path from x to y. For fixed $x^0 \in V$ we set

$$W_n = \{x \in V | d(x, x^0) = n\}, \quad V_n = \cup_{m=1}^n W_m.$$

Denote

$$S(x) = \{y \in W_{n+1} : d(x, y) = 1\}, \quad x \in W_n,$$

this set is called a set of direct successors of x.

Two vertices $x, y \in V$ is called *one level next-nearest-neighbor vertices* if there is a vertex $z \in V$ such that $x, y \in S(z)$, and they are denoted by $> x, y <$.

In this paper we consider the Potts model with competing interactions, where the spin takes values in the set $\Psi = \{\sigma_1, \sigma_2, ..., \sigma_q\}$, where $\sigma_1, \sigma_2, ..., \sigma_q$ are certain elements of $\mathbb{Q}_p^{q-1}$, on the Cayley tree which is defined by the following Hamiltonian

$$H(\sigma) = -\sum_{>x,y<} J_{xy} \delta_{\sigma(x),\sigma(y)} - \sum_{<x,y>} J_{1,xy} \delta_{\sigma(x),\sigma(y)}$$

where $J_{xy}, J_{1,xy} \in \mathbb{Q}_p$ are coupling constants and σ a configuration on V, i.e. $\sigma \in \Omega = \Psi^V$.

Continuing our investigates (see [1-2]) devoted to the phase transitions problems for the p-adic Potts models, we study the p-adic Potts model with competing interactions on the Cayley tree of order 2. It is proved that the existence of a phase transition for the model.

[1] F.M.Mukhamedov and U.A.Rozikov, On Gibbs measures of p-adic Potts model on the Cayley tree. *Indag. Mathem. N.S.* **15**(2004), 85-100.

[2] F.M.Mukhamedov and U.A.Rozikov, On inhomogeneous p-adic Potts model on a Cayley tree. *Infin. Dim. Anal. Quantum Probab. Rel. Topics* **8**(2005).