Proof of the Kurlberg-Rudnick rate conjecture

Shamgar Gurevich

Abstract

This is a joint work with **Ronny Hadani** under the supervision of Prof. Joseph Bernstein.

Consider the two dimensional symplectic torus $(\mathbf{T}, \omega)$ and an hyperbolic automorphism A of $\mathbf{T}$. The automorphism A is known to be ergodic. In 1980, using a non-trivial procedure called quantization, the physicists M.V. Berry and J. Hannay attached to this automorphism a quantum operator $\rho_\hbar(A)$ acting on a Hilbert space $\mathcal{H}_\hbar$. One of the central questions of "Quantum Chaos Theory", in this model, is whether the operator $\rho_\hbar(A)$ is "quantum ergodic"?

We consider the following two distributions on the algebra $\mathcal{A} = \mathcal{C}^\infty(\mathbf{T})$ of smooth complex valued functions on $\mathbf{T}$. The first one is given by the *Haar* integral:

$$f \longmapsto \int_{\mathbf{T}} f\omega$$

and the second one is given by the *Wigner* distribution:

$$f \longmapsto \mathcal{W}_\chi(f)$$

defined as the expectation of the "quantum observable" $\pi_\hbar(f)$ in the Hecke state v_χ, i.e. $\mathcal{W}_\chi(f) := \langle v_\chi | \pi_\hbar(f) v_\chi \rangle$. Here the vector v_χ is a common eigenvector, with eigencharacter χ, of the *Hecke* group of symmetries of the quantum operator $\rho_\hbar(A)$.

The *Kurlberg-Rudnick* rate conjecture is a quantitative description of the behavior of the Wigner distribution attached to the ergodic automorphism A. It states that for *Planck* constant of the form $\hbar = \frac{1}{p}$, where p is a prime number, one has:

Rate Conjecture. The following bound holds:

$$\left| \mathcal{W}_\chi(f) - \int_{\mathbf{T}} f\omega \right| \leq \frac{C_f}{\sqrt{p}}$$

where C_f is a constant that depends only on the function f.

In the current lecture we will present a proof of the Kurlberg-Rudnick conjecture. This is carried out using new representation theoretic constructions and algebro-geometric sheaf realization of the Weil metaplectic representation, which was proposed by P. Deligne in 1982 (see **arXiv:math-ph/0404074**).